Ch2

قانون كولوم

القوة الناتجة من شحنة Q على شحنة أخرى Q_2 تحدد من القانون

2nd
- point
- ∞ Line
- ∞ surface (Sheet)
- other form

$$\boxed{F_{12} = \frac{Q_1 Q_2}{4\pi \varepsilon_0 R^2} \bar{a}_R}$$

Q_1 $\bar{R}$ Q_2

P_1 P_2

R: نحدد المتجه

موضع النقطة الأولى P_1 وموضع النقطة الثانية P_2

$$\bar{R} = \bar{P}_2 - \bar{P}_1 = (x_{P_2} - x_{P_1})\bar{a}_x + (y_{P_2} - y_{P_1})\bar{a}_y + (z_{P_2} - z_{P_1})\bar{a}_z$$

where $|R| = \sqrt{(x_{P_2} - x_{P_1})^2 + (y_{P_2} - y_{P_1})^2 + (z_{P_2} - z_{P_1})^2}$

$\bar{a}_R = \frac{\bar{R}}{|R|}$ $\quad \varepsilon_0 = 8.85 \times 10^{-12} = \frac{10^{-9}}{36\pi}$

Electric field intensity ($\bar{E}$)

يكون المجال الكهربي عند نقطة P_2 الشحنات

إذا أثرت شحنة Q على وحدة الشحنات يحدد من العلاقة الآتية

الناتج من الشحنة Q

$$\boxed{E = \frac{Q}{4\pi \varepsilon_0 R^2} \bar{a}_R}$$

وبالتالي إذا وجدت شحنة Q_t عند الموقع

فإن القوة عليها تساوي $(Q)_{P_1}$ $\bar{R}$ P_2

$\boxed{F = QE}$ → بشكل عام → $\boxed{\bar{F}_Q = Q\,\bar{E}_{others}}$ القوة على شحنة

(3) هناك أشكال مختلفة من الشحنات (شحنة طولية - شحنة سطحية - شحنة حجمية)
و لحساب المجال الناتج من هذه الشحنات نتبع الخطوات الآتية

(a) نقسم الشحنة إلى أجزاء صغيرة (dQ) و تكون على شكل من الأشكال الآتية

$dQ = \rho_L \, dl$	$dQ = \rho_s \, ds$	$dQ = \rho_v \, dV$
حيث ρ_L الكثافة الطولية للشحنة ب (C/m) ويمكن أن يكون كالآتي $(dx, dy, dz, d\rho, r d\theta)$	حيث ρ_s كثافة الشحنة السطحية وتقاس ب (C/m^2) ويمكن أن يكون ds بأي من الأشكال الموجودة في الباب الأول	حيث ρ_v كثافة الشحنة الحجمية وتقاس ب (C/m^3) وحسب شكل الشحنة نستخدم dV

(b) نحسب المجال الناتجة من dQ عن طريق قانون كولوم

$$dE = \frac{dQ}{4\pi\varepsilon_0 R^2} \, a_R$$

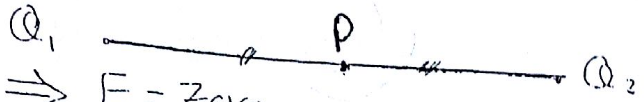
(c) نحدد قيمة R و بعد ذلك نكامل على أبعاد الشحنة حسب نوع

$$E = \int_{\substack{Vol. \\ Surface \\ Length}} dE = \int \frac{dQ}{4\pi\varepsilon_0 R^2} \, a_R$$

(d) من الأفضل جعل التكامل على صورة من الصورة الأسهل مثل التكامل على:

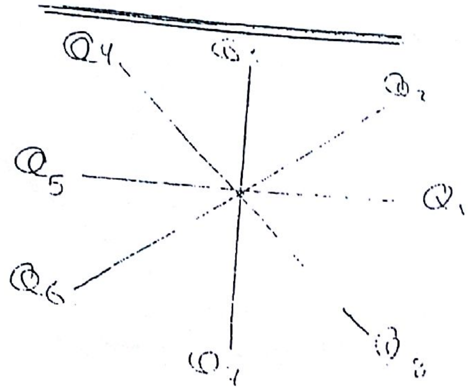
$dr, \, r d\theta, \, r\sin\theta \, d\phi$	$d\rho, \, \rho d\phi, \, dz$	dx, dy, dz	بدلا من أطوال
$r \rightarrow dr$	$r \rightarrow dr$	$x \rightarrow dx$	تذكر
$\theta \rightarrow r d\theta$	$\phi \rightarrow r d\phi$	$y \rightarrow dy$	
$\phi \rightarrow r\sin\theta \, d\phi$	$z \rightarrow dz$	$z \rightarrow dz$	

(1)



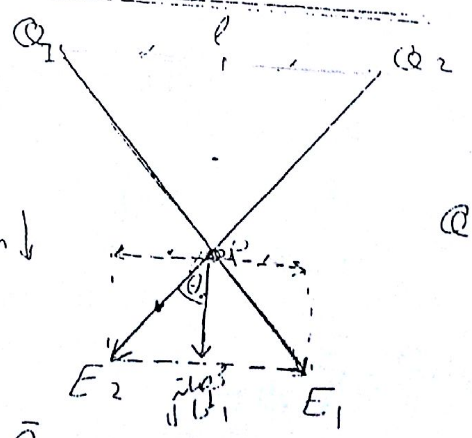
$Q_1 = Q_2 \Rightarrow E = Zero$

(2)



$Q_1 = Q_2 = Q_3 = \dots = Q_8 \Rightarrow E_p = Zero$

(3)



$Q_1 = Q_2 = Q$

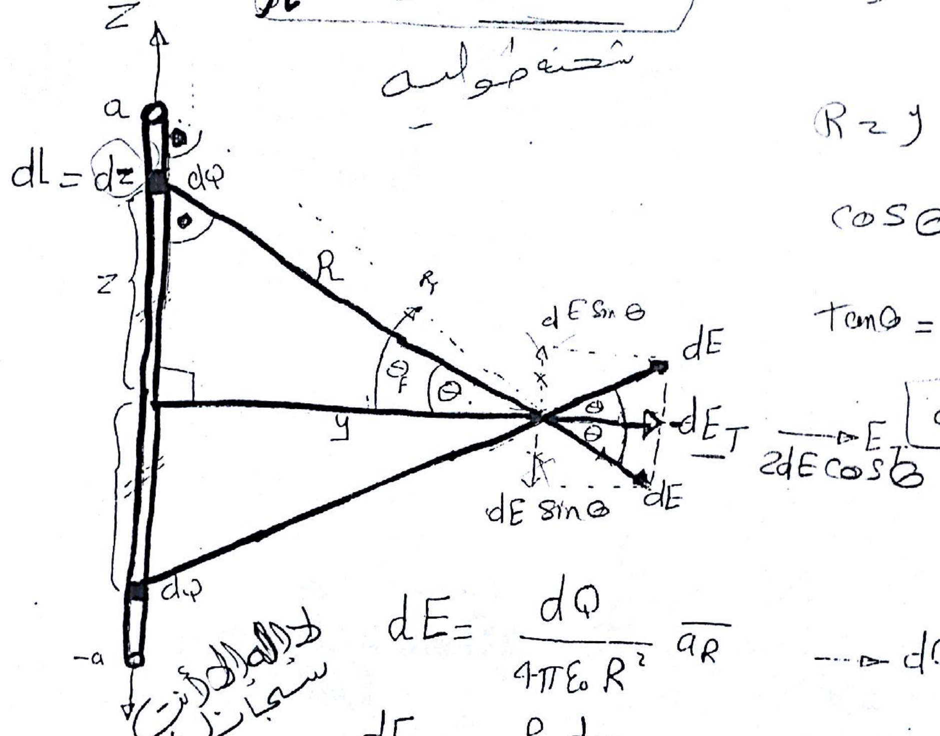
$$E_t = 2 \frac{Q}{4\pi \varepsilon_0 R^2} \cos\theta \cdot \bar{a}_m$$

* في حالة التماثل لا تبقى إلا المحصلة الرأسية و المحصلة الأفقية = صفر

lectric field intensity due to Line charge (1.3)

الخط مشحون بشحنة منتظمة على الطول ρ_L (C/m)

شحنه طوليه



$R = y\sec\theta$

$\cos\theta = \dfrac{y}{R}$

$\tan\theta = \dfrac{z}{y}$

$dz = y\sec^2\theta$

$$dE = \frac{dQ}{4\pi\varepsilon_0 R^2}\,\overline{a_R} \quad \longrightarrow dQ = \rho_L\, dL = \rho_L\, dz$$

$$dE = \frac{\rho_L\, dz}{4\pi\varepsilon_0 R^2}\,\overline{a_R}$$

نتيجة تأثير dQ و المتناظره على نفس المسافة Z ينتج مجال dE_T في اتجاه y وتتلاشى مركبة المجال الرأسى

$$dE_T = \frac{2\rho_L\, dz}{4\pi\varepsilon_0 R^2}\cos\theta\,\overline{a_y}$$

هكذا سيكون التكامل على نصف الخط فقط.

$$\boxed{dE_T = \frac{\rho_L\, dz\cos\theta}{2\pi\varepsilon_0 R^2}\,\overline{a_y}}$$

من الأفضل جعل التكامل على θ بدلاً من Z بمعنى من الأفضل تحويل كل ما يعتمد على Z في التكامل من Z إلى θ مثل (R ,dz) ونجعل كلاً منهما بدلالة θ وكذلك أى متغيرات ثابتة أخرى مثل y

$$\tan\theta = \frac{z}{y} \longrightarrow z = y\tan\theta \longrightarrow \boxed{dz = y\sec^2\theta\, d\theta}$$

$$\cos\theta = \frac{y}{R} \longrightarrow R = \frac{y}{\cos\theta} \longrightarrow \boxed{R^2 = y^2\sec^2\theta}$$

$$dE_T = \frac{\rho_L \cos\theta\, y \sec^2\theta \, d\theta}{2\pi\varepsilon_0 \, y^2 \sec^2\theta} \, \overline{a_y}$$

$$dE_T = \frac{\rho_L \cos\theta \, d\theta}{2\pi\varepsilon_0 \, y} \, \overline{a_y}$$

$$E_T = \int_{\theta=0}^{\theta_f} dE_T = \int_{\theta=0}^{\theta_f} \frac{\rho_L \cos\theta \, d\theta}{2\pi\varepsilon_0 \, y} \, \overline{a_y}$$

$$= \frac{\rho_L}{2\pi\varepsilon_0 y} \sin\theta \Big|_0^{\theta_f} \, \overline{a_y}$$

$$\frac{\rho_L}{2\pi\varepsilon_0 y} \sin\theta$$

$$= \frac{\rho_L}{2\pi\varepsilon_0 y} \sin\theta_f \, \overline{a_y}$$

مشروق

$$\sin\theta_f = \frac{a}{R_f}$$

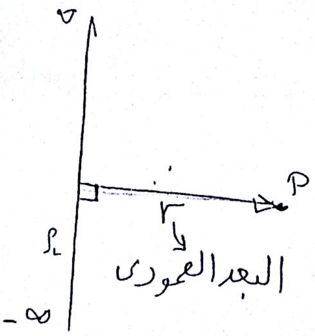
if the line is $\boxed{\text{infinite line}}$ $\theta_f \longrightarrow \frac{\pi}{2}$

$$\therefore E_T = \frac{\rho_L}{2\pi\varepsilon_0 y} \, \overline{a_y}$$

للشكل ى

العمودي

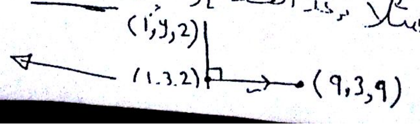
$$\boxed{\bar{E}_T = \frac{\rho_L}{2\pi\varepsilon_0 r} \, \overline{a_r}}$$



r: في حالة الخط هي البعد العمودي بين الخط و P ولحساب المتجة r نحسبه من البعدين المعرف فيهما

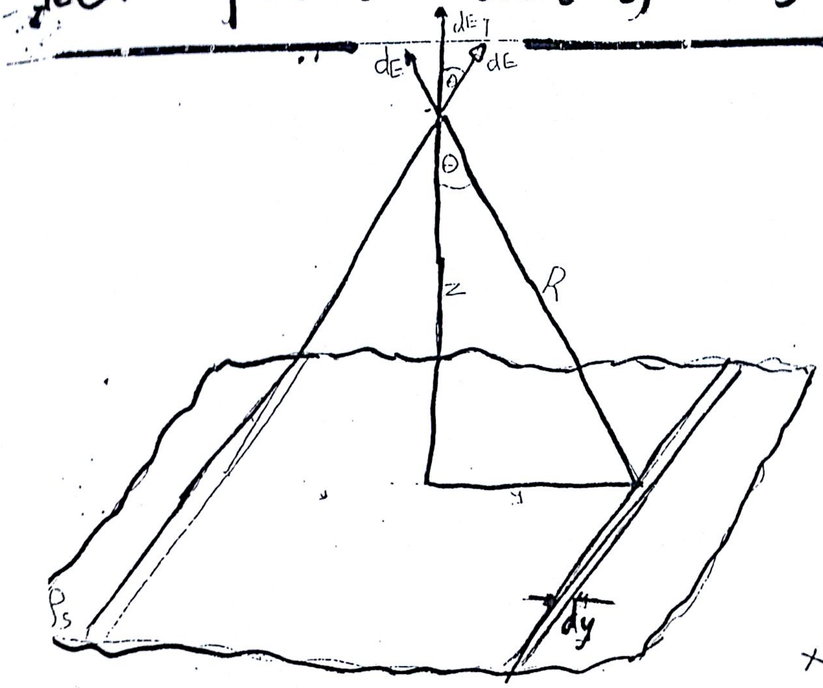
مثلا يوجد الخط ρ_L الموجود عند $x=1$, $z=2$ عن النقطة P $(9,3,9)$

$$\bar{r} = 8\,\overline{a_x} + 7\,\overline{a_z}$$



Electric field due sheet of charges

(٣)



$$dE = \frac{\rho_L}{2\pi\varepsilon_0 R}\, a_R$$

كثافة الشحنة السطحية ρ_s

لحساب المجال الناتج من sheet نقسمه إلى مجموعة من الخطوط كل خط له dy وبالتالي تكون قيمة (ρ_L) كالاتي

$$Q = \rho_L \, dL = \rho_s \, dL \, dy$$

$$\rho_L = \rho_s \, dy$$

المجال عند الشريحة = مجال عند خط لانهائي

$$dE = \frac{\rho_L}{2\pi\varepsilon_0 R}\, a_R$$

$$dE = \frac{\rho_s \, dy}{2\pi\varepsilon_0 R}\, \overline{a_R}$$

بتحليل dE في الأفقي والرأسي z نتبقى المركبة للمجال في اتجاه (z)

$$dE_T = \boxed{2\, dE \cos\theta}$$

$$dE_T = \frac{2\rho_s \cos\theta \, dy}{2\pi\varepsilon_0 R}\, \overline{a_z}$$

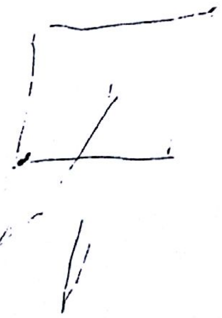
$$dE_T = \frac{\rho_s \cos\theta \, dy}{\pi\varepsilon_0 R}\, \overline{a_z}$$

(٢) , يجب تحويل كلا من (dy) و (R) المتغيرات إلى ... θ ...

$$\tan\theta = \frac{y}{Z} \rightarrow y = Z\tan\theta \longrightarrow dy = Z\sec^2\theta\, d\theta$$

$$\cos\theta = Z/R \rightarrow R = \frac{Z}{\cos\theta} \rightarrow R = Z\sec\theta$$

$$dE_T = \frac{\rho_s \cos\theta * \cancel{Z}\sec^2\theta\, d\theta}{\pi\varepsilon_0\, \cancel{Z}\sec\theta}\, \overline{a_z}$$



$$dE_T = \frac{\rho_s\, d\theta}{\pi\varepsilon_0}\, \overline{a_z}$$

$$E_T = \int_{\theta=0}^{\frac{\pi}{2}} dE_T = \int_0^{\frac{\pi}{2}} \frac{\rho_s}{\pi\varepsilon_0}\, d\theta\, \overline{a_z}$$

$$= \frac{\rho_s}{\pi\varepsilon_0}\, \theta\Big|_0^{\frac{\pi}{2}} \overline{a_z} = \frac{\rho_s}{2\varepsilon_0}\, \overline{a_z}$$

المجال الناتج من المستوى اللانهائي لا يعتمد على بعد النقطة المؤثر عليها من المستوى ولكن من الممكن أن يكون المجال + أو − حسب تواجد النقطة؛ المستوى مثلا

مستوى عند $X=2$ والنقطة أبعادها $(\overset{x}{3}, \overset{y}{4}, \overset{z}{5})$ ∴ المجال في اتجاه $(+\overline{a_x})$

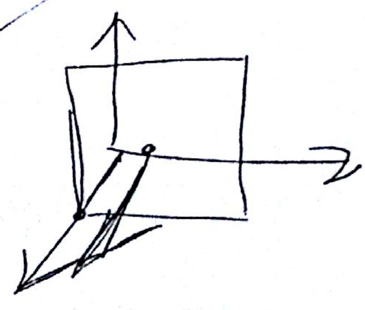
" " $Y=4$ والنقطة عند $(\overset{x}{2}, \overset{y}{0}, \overset{z}{3})$ ∴ المجال في اتجاه $(-\overline{a_y})$

" " $Z=2$ " " $(\overset{x}{2}, \overset{y}{3}, \overset{z}{7})$ ∴ " " " $(+\overline{a_z})$

بشكل عام

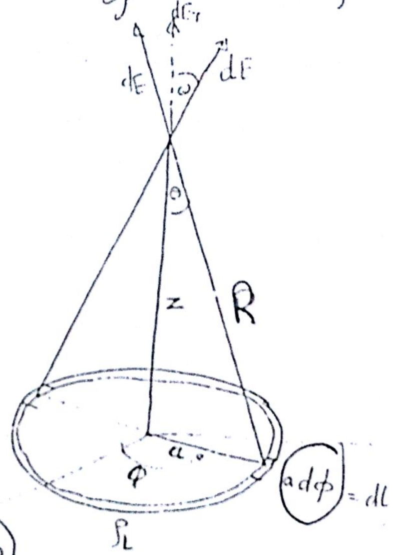
$$\boxed{\bar{E} = \frac{\rho_s}{2\varepsilon_0}\, a_N}$$

الاتجاه العمودي على السطح



The electric field intensity due to ring of charges (7)

Find the electric field intensity produced by ring of charges and find the position of maximum field value and the value of maximum field



$$dQ = \rho_L\, dl = \rho_L\, a\, d\phi$$

$$dE = \frac{dQ}{4\pi\varepsilon_0 R^2}\,a_R$$

$$dE = \frac{\rho_L\, a\, d\phi}{4\pi\varepsilon_0 R^2}\,\bar{a}_R$$

$$dE_T = \frac{2\rho_L\, a \cos\theta\, d\phi}{2\cdot 4\pi\varepsilon_0 R^2}\,\bar{a}_z$$

$$\cos\theta = \frac{z}{R}$$

$$dE_T = \frac{\rho_L\, a\, z\, d\phi}{2\pi\varepsilon_0 R^3}\,\bar{a}_z$$

$$E_T = \int_{\phi=0}^{\pi} dE_T = \frac{\rho_L\, a\, z}{2\pi\varepsilon_0 R^3}\int_{\phi=0}^{\pi} d\phi$$

$$= \frac{\rho_L\, a\, z}{2\pi\varepsilon_0 R^3} * \pi = \frac{\rho_L\, a\, z}{2\varepsilon_0 (z^2 + a^2)^{3/2}}$$

(ا) يلاحظ أن المجال الناتج من ~~[illegible]~~ الحلقة يساوي صفر عند $Z=0$
وكذلك عند $Z=\infty$ وبالتالي عند قيمة معينة يصل إلى أقصى قيمة:
قيمة بينهما وللحصول عليها نفاضل E_T بالنسبة إلى Z ونساوي بالصفر

$$0 = \frac{dE_T}{dz} = \frac{\rho_L a}{2\varepsilon_0}\left[\frac{(Z^2+a^2)^{\frac{3}{2}} - Z * \frac{3}{\cancel{2}}(Z^2+a^2)^{\frac{1}{2}} * \cancel{2}Z}{(Z^2+a^2)^3}\right]$$

$$0 = (Z^2+a^2)^{\frac{3}{2}} - 3Z^2(Z^2+a^2)^{\frac{1}{2}}$$

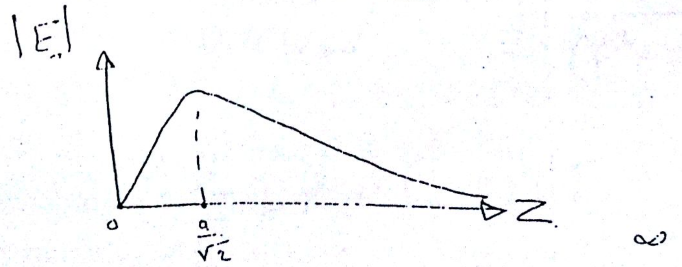
$$0 = (Z^2+a^2)^{\frac{1}{2}}\left[(Z^2+a^2) - 3Z^2\right]$$

لا يساوي صفر أبدا

$$Z^2 + a^2 - 3Z^2 = 0$$

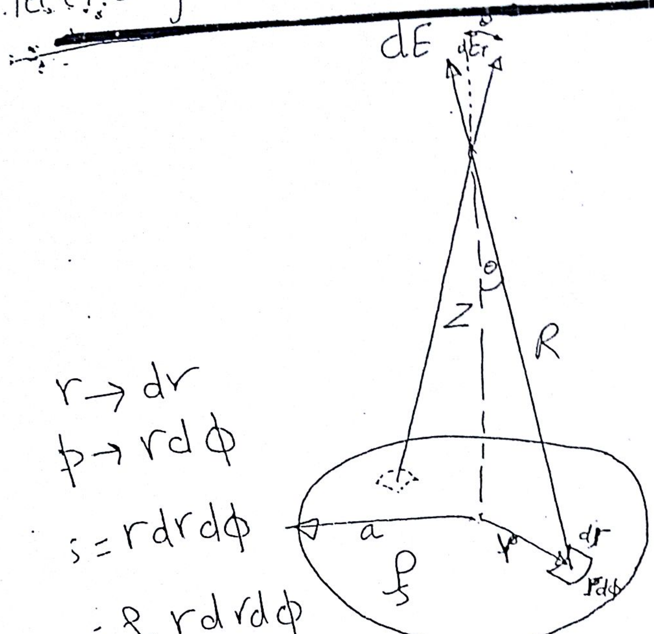
$$2Z^2 = a^2$$

موضع أقصى مجال $\boxed{Z = \pm \frac{a}{\sqrt{2}}}$



$$E_T\Big|_{max} = \frac{\rho_L a * \frac{a}{\sqrt{2}}}{2\varepsilon_0\left(\left(\frac{a}{\sqrt{2}}\right)^2 + a^2\right)^{\frac{3}{2}}} = \frac{\rho_L a^2}{2\sqrt{2}\,\varepsilon_0\left(\frac{3}{2}a^2\right)^{\frac{3}{2}}}$$

Electric field due to disk of charges



$r \to dr$

$\phi \to r\,d\phi$

$s = r\,dr\,d\phi$

$= \rho_s\, r\,dr\,d\phi$

$$dE = \frac{dQ}{4\pi\varepsilon_0 R^2}\, a_R$$

$$dE = \frac{\rho_s\, dS}{4\pi\varepsilon_0 R^2}\, \bar{a}_R$$

$$= \frac{\rho_s\, r\, d\rho\, d\phi}{4\pi\varepsilon_0 R^2}\, \bar{a}_R$$

$$dE_T = 2\, dE \cos\theta\ \bar{a}_z$$

$$dE_T = \frac{\rho_s\, r\, dr\, d\phi}{2\pi\varepsilon_0 R^2} \cos\theta\ a_z$$

$$\boxed{\cos\theta = \frac{z}{R} \quad R = \sqrt{\rho^2+z^2}}$$

$$dE_T = \frac{\rho_s}{2\pi\varepsilon_0}\, z\, \frac{\rho\, d\rho\, d\phi}{R^3}\ \bar{a}_z$$

$$E_T = \int_{\phi=0}^{\pi}\int_{\rho=0}^{a} dE_T = \frac{\rho_s z}{2\pi\varepsilon_0} \int_{\phi=0}^{\pi} d\phi \cdot \int_{0}^{a} \frac{r\, dr}{(r^2+z^2)^{\frac{3}{2}}}\ a_z$$

$$= \frac{\rho_s z}{2\pi\varepsilon_0} * \cancel{\pi}\ \frac{1}{2} \int_{0}^{a} \frac{2\rho\, d\rho}{(\rho^2+z^2)^{\frac{3}{2}}}\ a_z$$

$$= \frac{\rho_s z}{2\varepsilon_0} \cdot \cancel{\frac{1}{2}} \cdot -\cancel{2}\ \frac{1}{(\rho^2+z^2)^{\frac{1}{2}}}\Bigg|_0^a\ a_z$$

$$= \boxed{\frac{\rho_s z}{2\varepsilon_0}\left[\frac{1}{z} - \frac{1}{(a^2+z^2)^{\frac{1}{2}}}\right]\bar{a}_z}$$

$a \to \infty$ disk $\to$ infinite sheet

$E = \frac{\rho_s}{2\varepsilon_0}\, a_z$

Field due to infinite sheet

يتم اثبات حالة disk حتى نصل
إلى القانون

$$E = \frac{\rho_s Z}{2\varepsilon_0}\left[\frac{-1}{\sqrt{a^2+z^2}} + \frac{1}{z}\right]$$

ثم نقول التربيع infinite sheet $\Leftarrow$ $a = \infty$

$$\therefore \; E = \frac{\rho_s Z}{2\varepsilon_0}\left[\frac{1}{\infty} + \frac{1}{z}\right] a_N$$

$$\boxed{E = \frac{\rho_s}{2\varepsilon_0} a_N} \;\#$$

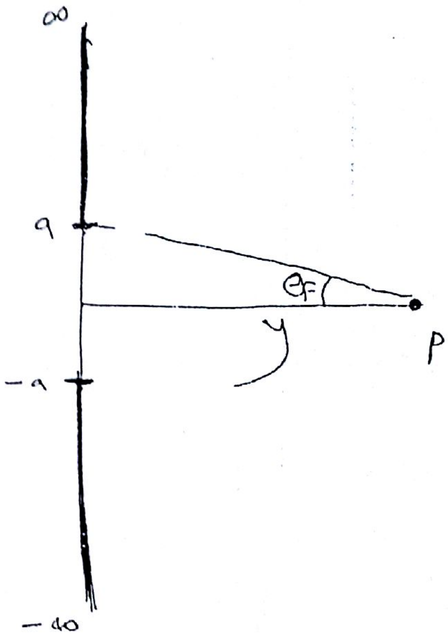
ملاحظات ① المجال عند سطح لانهائي لا يعتمد على النقطة
فهو ثابت في القيمة مهما كانت النقطة

② a_N قد يكون ⊕ أو ⊖ وذلك
يتوقف على موضع النقطة بالنسبة للسطح

line charge

لكي نجيب المجال عند خط
نهائي

$a \rightarrow \infty$
$-\infty \rightarrow -a$



لازم كالآتي

$$E = \underbrace{E}_{\text{Infinite line}} - \underbrace{E}_{-a \to a}$$

$\downarrow$ $\frac{\rho_L}{2\pi\varepsilon_0 y}$

$\downarrow$ نعمل خطوات الاثبات لخط محدود $\downarrow$ $\frac{\rho_L}{2\pi\varepsilon_0 y} \sin\theta_f$

$$\therefore \boxed{E = \frac{\rho_L}{2\pi\varepsilon_0 y}\left\{1 - \sin\theta_f\right\}}$$

$$\sin\theta_f = \frac{a}{\sqrt{a^2+y^2}} \quad \#$$

example

Line charge $\rho_L = 20\ nC/m$

extend from $z = 5 \rightarrow \infty$

and from $z = -5 \rightarrow -\infty$

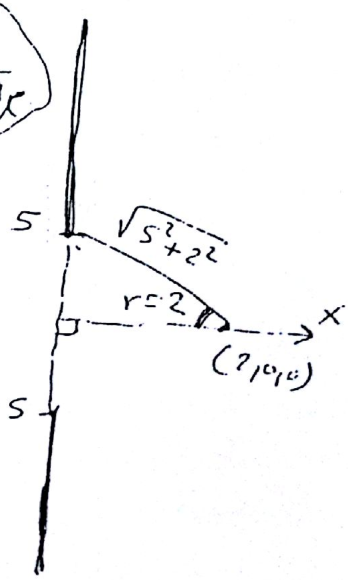
Find $\bar{E}$ at $(2,0,0)$

solution

$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\, a_r - \int_{-5}^{5} \frac{dQ}{4\pi\varepsilon_0 R^2}\, \bar{a}_R$$

انتاج خط محدود الطول

نحن الي



$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\left[1 - \sin\theta_f\right]$$

$r = 2$

$$\sin\theta_f = \frac{5}{\sqrt{5^2+2^2}}$$

$$\bar{E} = 12.86\ \bar{a}_x\ \ V/m.$$

Find the relationship which the cartesian components of **A** and **B** must satisfy if the vector fields are everywhere parallel.

$\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$, $\bar{B} = B_x \bar{a}_x + B_y \bar{a}_y + B_z \bar{a}_z$

$\bar{A}$ parallel with $\bar{B}$ $\rightarrow$ $\therefore$ $\bar{A} \times \bar{B} = $ Zero

$$\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = 0 \rightarrow \begin{aligned} &(A_y B_z - A_z B_y)\bar{a}_x \\ &-(A_x B_z - A_z B_x)\bar{a}_y = 0 \\ &+(A_x B_y - A_y B_x)\bar{a}_z = \end{aligned}$$

$A_y B_z = A_z B_y \rightarrow \frac{A_y}{B_y} = \frac{A_z}{B_z}$

$\rightarrow A_x B_z = A_z B_x \rightarrow \frac{A_x}{B_x} = \frac{A_z}{B_z}$

$\therefore \boxed{\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}}$ //

Transform

$$\mathbf{A} = y\mathbf{a}_x + x\mathbf{a}_y + \frac{x^2}{\sqrt{x^2+y^2}}\mathbf{a}_z$$

from cartesian to cylindrical coordinates.

$\rightarrow$ result $\rightarrow$ $\bar{A} = 2r \sin\phi \cos\phi \, \bar{a}_r + (r\cos^2\phi - r\sin^2\phi)\bar{a}_\phi + r\cos^2\phi \, \bar{a}_z$

Use the spherical coordinate system to find the area of the strip $\alpha \le \theta \le \beta$ on the spherical shell of radius a (Fig. 1-11). What results when $\alpha = 0$ and $\beta = \pi$?

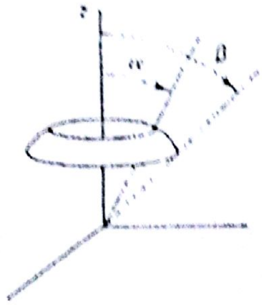


Fig. 1-11

$$dS = r^2 \sin\theta \, d\theta \, d\phi$$

Then

$$A = \int_0^{2\pi}\int_\alpha^\beta a^2 \sin\theta \, d\theta \, d\phi$$
$$= 2\pi a^2(\cos\alpha - \cos\beta)$$

When $\alpha = 0$ and $\beta = \pi$, $A = 4\pi a^2$, the surface area of the entire sphere.

$0<\phi<\pi \rightarrow Q_1$

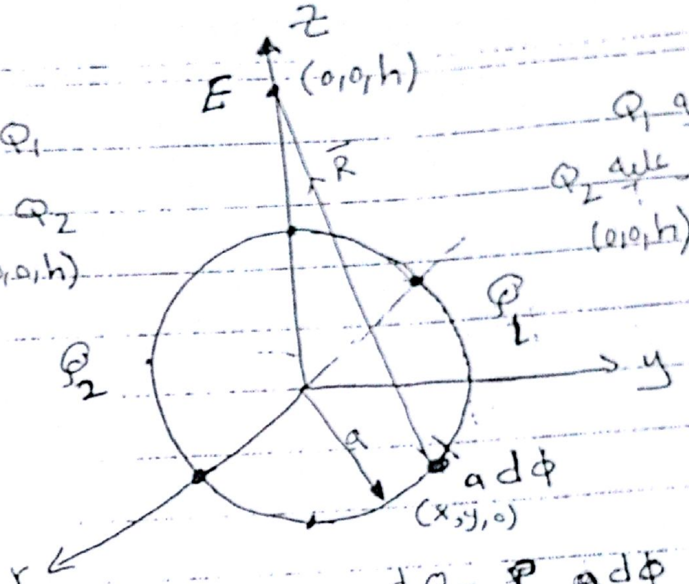
$\pi<\phi<2\pi \rightarrow Q_2$

Find $\bar{E}$ at $(0,0,h)$

حالة نصفها عليه Q_1

والنصف الآخر عليه Q_2

احسب المجال عند $(0,0,h)$



$dQ_1 = \rho_{L_1}\, a\, d\phi$

Note

$\rho_{L_1} = \frac{Q_1}{\pi a}$

$\rho_{L_2} = \frac{Q_2}{\pi a}$

$0<\phi<\pi$

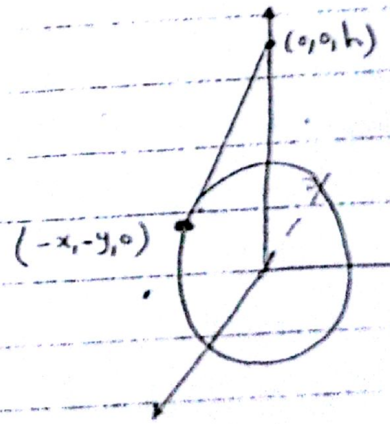
$$dE_1 = \frac{dQ_1}{4\pi\varepsilon_0 R_1^2}\,\bar{a}_{R_1}$$

$$\bar{R}_1 = (0,0,h) - (x,y,0) = (-x\,\bar{a}_x - y\,\bar{a}_y) + h\,\bar{a}_z$$
$$= -a\,\bar{a}_r + h\,\bar{a}_z$$

$$|R| = \sqrt{a^2+h^2}$$

$$E_1 = \int dE_1 = \int_0^{\pi} \frac{\rho_{L_1}\, a\, d\phi}{4\pi\varepsilon_0 (a^2+h^2)} \frac{-a\,\bar{a}_r + h\,\bar{a}_z}{\sqrt{a^2+h^2}}$$

$$\bar{E}_1 = \frac{\rho_{L_1}\, a}{4\varepsilon_0 (a^2+h^2)^{3/2}}(-a\,\bar{a}_r + h\,\bar{a}_z) \rightarrow (1)$$



$\pi<\phi<2\pi$

$$\bar{R}_2 = (0,0,h) - (-x,-y,0)$$
$$= x\,\bar{a}_x + y\,\bar{a}_y + h\,\bar{a}_z$$
$$= a\,\bar{a}_r + h\,\bar{a}_z$$

$$dQ_2 = \rho_{L_2}\, a\, d\phi$$

$$d\bar{E}_2 = \frac{dQ_2}{4\pi\varepsilon_0 R_2^2}\,\bar{a}_{R_2}$$

$$E_2 = \int^{2\pi} dE_2 = \frac{\rho_{L2}\, a}{(\ldots)^{3/2}}(a\,\bar{a}_r + h\,\bar{a}_z) \rightarrow (2)$$

$$\boxed{E = E_1 + E_2}$$ #

$$\bar{E} = \frac{\rho_{L_1} a}{4\varepsilon_0 (a^2+h^2)^{3/2}} (-a_1 \bar{a}_r + h \bar{a}_z) + \frac{\rho_{L_2} a}{4\varepsilon_0 (a^2+h^2)^{3/2}} (a \bar{a}_r + h \bar{a}_z)$$

if $\rho_{L_1} = \rho_{L_2} \rightarrow Q_1 = Q_2$

$$\bar{E} = \frac{\rho_L a h}{2\varepsilon_0 (a^2+h^2)^{3/2}} \bar{a}_z$$

1. A 20 nC point charge is located at P (2, 4, -3) in free space. **Find**
   (a) E (r).
   (b) E at A (-3,2,0).

2. Point charges $Q_1 = 5\ \mu C$ and $Q_2 = -4\ \mu C$ are placed at (3, 2, 1) and (-4, 0, 6), respectively.
   Determine the force on Q_1.

3. Point charges Q_1 and Q_2 are, respectively, located at (4, 0, -3) and (2, 0, 1). If $Q_2 = 4$ nC, **Find** Q_1 such that
   (a) The E at (5, 0, 6) has no z-component
   (b) The force on a test charge at (5, 0, 6) has no x-component.

4. **Determine** the total charge
   (a) On line $0 < x < 5$ m if $\rho_l = 12x^2$ mC/m
   (b) On the cylinder $\rho = 3$, $0 < z < 4$ m if $\rho_s = \rho z^2$ nC/m^2
   (c) Within the sphere $r = 4$ m if $\rho_v = \frac{10}{r \sin\theta}$ C/m^3

5. A line charge density ρ_l is uniformly distributed over a length of 2a with centre as origin along x axis. **Find E** at a point P which is on z axis at a distance d.

6. It is required to hold four equal point charges each in equilibrium at the corners of square. **Find** the point charge which will do this, if placed at the centroid of the square.

7. A ring placed along $y^2 + z^2 = 4$, $x = 0$ carries a uniform charge of 5 μC/m.
   (a) **Find** E at P (3,0, 0).
   (b) If two identical point charges Q are placed at (0, -3, 0) and (0, 3, 0) in addition to the ring,
   Find the value of Q such that E = 0 at P.

8. A sheet of charges $\rho_s = 2$ nC /m^2, is present at the plane x = 3 in free space, and a line charge $\rho_l = 2$ nC 1m 1 is located at x = 1, z = 4. Find
   (a) The magnitude of electric field intensity at the origin.
   (b) The direction of E at p (4, 5, 6).
   (c) What is the force per meter length on the line charge?

9. A point charge 100 pC is located at (4, 1,-3) while the x-axis carries charge 2 nC/m. If the plane z = 3 also carries charge 5 nC/m^2, **Find E** at (1, 1, 1).

SHEET 3

Q1:

(a): at any point (x, y, z)

$$\bar{E}_{(r)} = \frac{1}{4\pi\epsilon_0} \frac{Q}{|r|^2} \bar{a}_r = \frac{180}{r^2} \bar{a}_r$$

as a function in (r) — دالة في (r)

لو عايز تجيبها عند نقطة ما تبقى (x, y, z)

$$\bar{r} = (x-2, y-4, z+3)$$

$$|r| = \sqrt{(x-2)^2 + (y-4)^2 + (z+3)^2}$$

$$\therefore \bar{E} = 180 \frac{(x-2)\bar{a}_x + (y-4)\bar{a}_y + (z+3)\bar{a}_z}{\left[(x-2)^2 + (y-4)^2 + (z+3)^2\right]^{3/2}}$$

(b): نعوض في المعادلة السابقة بالنقطة $\overset{x}{(-3}, \overset{y}{2}, \overset{z}{0})$

$$\therefore \bar{E} = 180 \frac{-5\bar{a}_x - 2\bar{a}_y + 3\bar{a}_z}{\left[(-5)^2 + (-2)^2 + (3)^2\right]^{3/2}}$$

$$= -3.8421\,\bar{a}_x - 1.5368\,\bar{a}_y + 2.3053\,\bar{a}_z$$

Q2 :

$Q_1 = 5\,\mu C \qquad (3, 2, 1)$

$Q_2 = -4\,\mu C \qquad (-4, 0, 6)$

point charges:

$$F_{21} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r_{12}^2} \bar{a}_{r_{12}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r_{21}^3} \bar{r}_{21}$$

$$r_{21} = (3 - (-4), 2 - 0, 1 - 6)$$

$$= 7\bar{a}_x + 2\bar{a}_y - 5\bar{a}_z$$

$$|r_{21}| = \sqrt{(7)^2 + (2)^2 + (-5)^2} = \sqrt{318}$$

$$F_{21} = 9\times10^9 \frac{(5\times10^{-6})(-4\times10^{-6})}{(318)^{3/2}} \left[7\bar{a}_x + 2\bar{a}_y - 5\bar{a}_z\right]$$

$$= 2.2219\times10^{-4}\bar{a}_x + 6.348\times10^{-5}\bar{a}_y - 1.5871\times10 \,\bar{a}_z$$

N

Q3:

$Q_1 = ?$ $(4, 0, -3)$

$Q_2 = 4\ nC$ $(2, 0, 1)$

نوجد E عند النقطة $(5,0,6)$ بعد ذلك نساوي
مركبة z بالصفر .

$$\bar{E}_T = \bar{E}_1 + \bar{E}_2$$

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{r_1^3} \bar{r}_1$$

$\bar{r}_1 = (1, 0, 9)$, $|\bar{r}_1| = \sqrt{1 + 0 + 81} = \sqrt{82}$

$$E_1 = 9 \times 10^9 \frac{Q_1}{(82)^{3/2}} (\bar{a}_x + 9\bar{a}_z)$$

$$= Q_1 \left[12.121 \times 10^6\right] \bar{a}_x + Q_1 \left[109.085 \times 10^6\right] \bar{a}_z$$

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{Q_2}{r_2^3} \bar{r}_2$$

$\bar{r}_2 = (3, 0, 5)$, $|\bar{r}_2| = \sqrt{9 + 25} = \sqrt{34}$

$$\bar{E}_2 = 9 \times \cancel{10^9} \frac{4 \times \cancel{10^{-9}}}{(34)^{3/2}} (3\bar{a}_x + 5\bar{a}_z)$$

$$= 0.54476\ \bar{a}_x + 0.9079\ \bar{a}_z$$

$$\therefore E_T = \left[Q_1 (12.121 \times 10^6) + 0.54476 \right] \bar{a}_x$$

$$+ \left[Q_1 (109.085 \times 10^6) + 0.9079 \right] \bar{a}_z$$

z - Component = 0

$$\therefore Q_1 (109.085 \times 10^6) + 0.9079 = 0$$

$$\therefore Q_1 = - \frac{0.9079}{109.085 \times 10^6} = -8.323 \text{ n}$$

(b):

$$\therefore F_E = q E$$

نجعل أن القوه = صفر أي المجال سيساوي الصفر

$$\therefore Q_1 (12.121 \times 10^6) + 0.54476 = 0$$

$$\therefore Q_1 = - \frac{0.54476}{12.121 \times 10^6} = -44.945 \text{ n}$$

Q4 :

a)

$$Q = \int \rho_L \, dL = \int_0^5 \rho_L \, dx \quad \times 10^{-3}$$

mc/m

$$= \int_0^5 12x^2 \, dx = \cancel{12} \, \frac{x^3}{\cancel{3}} \Big|_0^5 = 4\,(5)^3 ,$$

$$= 500 \; C \times 10^{-3} = 0.5 \; C$$

b)

$$dQ = \rho_s \, ds = \rho \, z^2 \, \rho \, d\phi \, dz \times 10$$

محن $\rho = 3$ ← أخذ ثابتة فالشريحة بالصيغة

تكامل في z , ϕ

$$\therefore Q = \rho^2 \int_0^4 \int_{\phi=0}^{2\pi} z^2 \, d\phi \, dz \quad \times 10^{-9}$$

$$= (3)^{\cancel{2}} \; 2\pi \; \frac{z^3}{\cancel{3}} \Big|_0^4 = 6\pi \, (4)^3 \times 10^{-9}$$

$$= 1.206 \; \mu C$$

c)

$$dQ = \rho_v \, dv = \frac{10}{\cancel{r \sin\theta}} \; r^{\cancel{2}} \, \cancel{\sin\theta} \, d\theta \, d\phi \, dr$$

dr

$r\,d\theta$

$r \sin\theta \, d\phi$

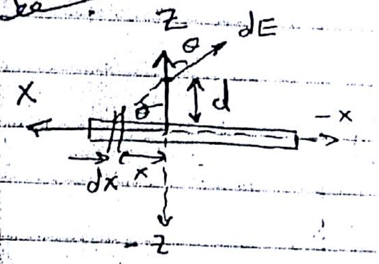
$$\therefore Q = \int_{r=0}^{4} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} r \, d\theta \, d\phi \, dr = \underbrace{\frac{(4)^2}{2}}_{r^2/2} (\pi)(2\pi)$$

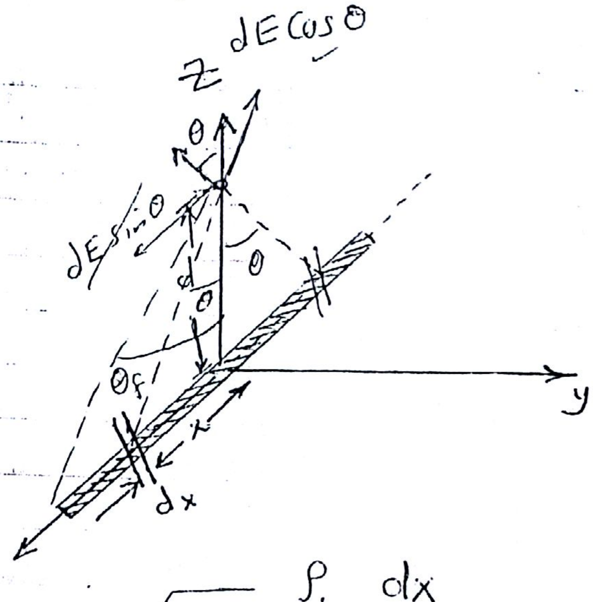
$$= 157.914 \; C$$

Q5 :

عكس تماثل

١٧ بقط





$$dE = \frac{1}{4\pi\epsilon_0} \frac{d\varphi}{r^2} \bar{a}_r \qquad d\varphi \leftarrow \rho_L \, dx$$

because of symmetry, the only component is at z-axis

$$\therefore E = \int 2\, dE \cos\theta$$

$$= \int \cancel{2} \frac{1}{_2\cancel{4}\pi\epsilon_0} \frac{\rho_L \, dx}{r^2} \cos\theta \; \bar{a}_z$$

$$\because \tan\theta = \frac{x}{d} \qquad \therefore x = d\tan\theta$$

$$dx = d \sec^2\theta \, d\theta$$

$$\because \cos\theta = \frac{d}{r} \qquad \therefore r = \frac{d}{\cos\theta} = d\sec\theta$$

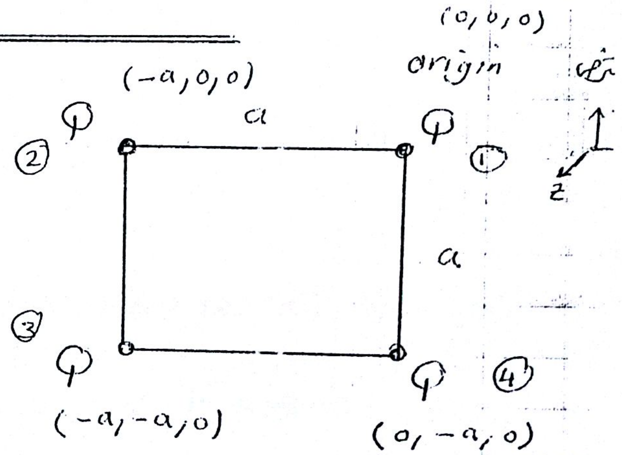
$$\therefore E = \frac{\rho_L}{2\pi\epsilon_0} \int_0^{\theta_f} \frac{(\cancel{d}\,\cancel{\sec^2}\theta \, d\theta)}{d^{\cancel{2}}\,\cancel{\sec^2}\theta} \cos\theta \; \bar{a}_z$$

$$= \frac{\rho_L}{2\pi\epsilon_0 d} \Big[\sin\theta \Big]_0^{\theta_f}$$

$$\bar{E} = \frac{\rho_L}{2\pi\epsilon_0 d} \underbrace{\sin\theta_f}_{\text{from figure}} \quad \bar{a}_z \qquad \sin\theta_f = \frac{a}{\sqrt{a^2+d^2}}$$

$$= \frac{\rho_L}{2\pi\epsilon_0 d} \cdot \frac{a}{\sqrt{a^2+d^2}} \quad \bar{a}_z$$

Q6 :



نحسب القوة على أي شحنة

ولكي تصبح متزنة لابد

أن تكون القوة الناشئة

من الشحنات الأخرى تلاشيهم

equilibrium $\sum \bar{F} = 0$

$\bar{r}_{21} = (a, 0, 0)$, $|\bar{r}_{21}| = a$.

$\bar{r}_{31} = (a, a, 0)$, $|\bar{r}_{31}| = a\sqrt{2}$

$\bar{r}_{41} = (0, a, 0)$, $|\bar{r}_{41}| = a$

$$\bar{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{Q_2 Q_1}{|r_{21}|^3} \bar{r}_{21} = \frac{Q^2}{4\pi\epsilon_0 a^{\cancel{3}\,2}} \cancel{a}\, \bar{a}_x$$

$$= \frac{Q^2}{4\pi\epsilon_0 a^2} \bar{a}_x$$

$$\bar{F}_{31} = \frac{Q^2}{4\pi\epsilon_0 (a\sqrt{2})^3} \cdot a(\bar{a}_x + \bar{a}_y) = \frac{Q^2}{4\pi\epsilon_0 a^2 (2)^{3/2}} (\bar{a}_x + \bar{a}_y$$

$$\overline{F}_{41} = \frac{Q^2}{4\pi\epsilon_o a^2} \quad \overline{a}_y$$

$$\therefore \overline{F}_T = \overline{F}_{21} + \overline{F}_{31} + \overline{F}_{41}$$

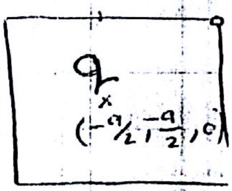
$$= \frac{Q^2}{4\pi\epsilon_o a^2} \Big[\overline{a}_x + (0.35355\,\overline{a}_x + 0.35355 + \overline{a}_y \Big]$$

$$= \frac{Q^2}{4\pi\epsilon_o a^2} \Big[1.35355\ \overline{a}_x + 1.35355\ \overline{a}_y \Big]$$

when we add the fifth charge at the Cen

at the equilibrium : $\Sigma \overline{F} = 0$ or,

$$\therefore \overline{F}_{51} = -\overline{F}_T$$



$$\overline{r}_{51} = \left(\frac{a}{2}, \frac{a}{2}, 0\right) \quad , |r_{51}| = \sqrt{\frac{a^2}{4} + \frac{a^2}{4}}$$

$$= \frac{a}{2}\sqrt{2}$$

$$F_{51} = \frac{Qq}{4\pi\epsilon_o \left(\frac{a}{2}\sqrt{2}\right)^3} \frac{a}{2} (\overline{a}_x + \overline{a}_y) = \frac{qQ}{4\pi\epsilon_o a^2} \frac{(2)}{(2)}$$

$$\therefore F_{51} = -F_T$$

$$\frac{q\cancel{Q}}{\cancel{4\pi\epsilon_o a^2}} \frac{4}{(2)^{3/2}} (\overline{a}_x + \overline{a}_y) = \frac{-Q^{\cancel{2}}}{\cancel{4\pi\epsilon_o a^2}} \Big[1.3536\ \overline{a}_x + 1.3$$

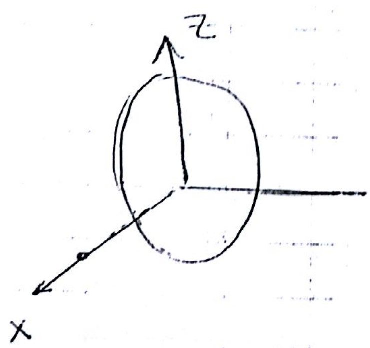
From x component (or y comp.)

$$q \frac{4}{(2)^{3/2}} = -Q \; 1.35355$$

$$\therefore \; q = -0.95711 \; Q$$

لا بد أن تكون إشارتها معاكسة للشحن Q حتى يحدث الاتزان .

Q7 :



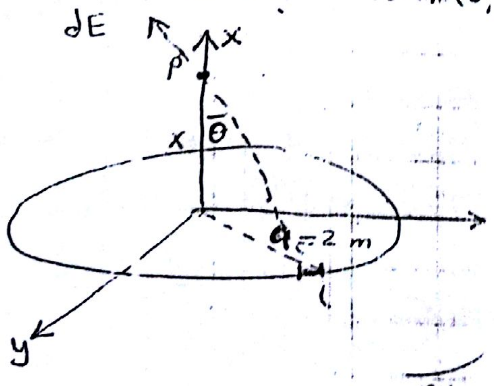
$$\underbrace{y^2 + z^2}_{\text{yz : يقع في المستوى}} = \overset{(radius)^2}{4}$$

وحوره على محور x

المحور عند x = 3

$\rho_L = 5 \times 10^{-6}$ c/m , $E = ?$ at P(3,

$$\therefore dE = \frac{1}{4\pi\epsilon_0} \frac{dQ}{|r^2|} \bar{a}_r$$



$$dQ = \rho_L \, dL = \rho_L \, a \, d\phi$$

because of symmetry :

صافيات نلاور

$$E = \int 2 \, dE \cos\theta \quad , \quad \bar{a}_r = \bar{a}_x$$

$$= \int \cancel{2} \frac{1}{\cancel{4}_{2}\pi\epsilon_0} \frac{\rho_L \, a \, d\phi}{(a^2 + x^2)} \cdot \frac{x}{\sqrt{x^2+a^2}} \; \bar{a}_x$$

$$\therefore E_{Ring} = \frac{\rho_L}{2\pi\epsilon_0} \frac{a\,x}{(a^2+x^2)^{3/2}} \int_0^{\pi} d\phi$$

$$= \frac{\rho_L}{2\epsilon_0} \frac{a\,x}{(a^2+x^2)^{3/2}} \; \bar{a}_x$$

أو بالتكامل :

$$E = \int 2\, dE \cos\theta$$

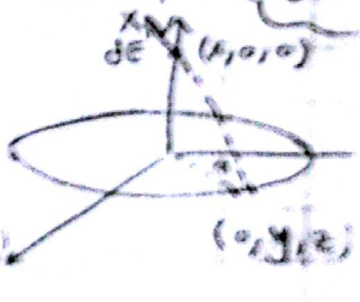
$$= \int 2 \frac{\rho_L\, a\, d\phi}{4\pi\epsilon_0\, r^2} \cos\theta \;\; \bar{a}_x$$

$$\because \cos\theta = \frac{x}{r}$$

$$\therefore E_{Ring} = \frac{\rho_L}{2\pi\epsilon_0} \frac{a\,x}{r^3} \int_0^{\pi} d\phi = \frac{\rho_L\, a\, x}{2\,\epsilon_0\, r^3} \; \bar{a}$$

$$= \frac{\rho_L\, a\, x}{2\,\epsilon_0\,(a^2+x^2)^{3/2}} \; \bar{a}_x$$

أو باستخدام الاحداثيات :



$$dE = \frac{1}{4\pi\epsilon_0} \frac{\rho_L\, a\, d\phi}{r^3} \; \bar{r}$$

$$\bar{r} = (x, -y, -z) = x\,\bar{a}_x \underbrace{(-y\,\bar{a}_y - z\,\bar{a}_z)}_{-a\,\bar{a}_r}$$

$$|r| = \sqrt{\underbrace{x^2}_{9} + \underbrace{y^2+z^2}_{a^2 = 4}} = \sqrt{13} \quad , \quad \bar{r} = x\,\bar{a}_x - \ldots$$

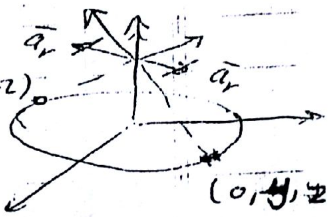
$$= \sqrt{x^2+a^2}$$

$$\therefore \quad dE = \frac{1}{4\pi\epsilon_0} \frac{\rho_L \, a \, d\phi}{r^3} \bar{r}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\rho_L \, a}{(x^2+a^2)^{3/2}} (x\,\bar{a}_x - a\,\bar{a}_r)$$

because of symmetry

the $\bar{a}_r$ (radial) component will c each other



$\bar{r} = (x, y, z)$

$= x\,\bar{a}_x + a\,\bar{a}_r$

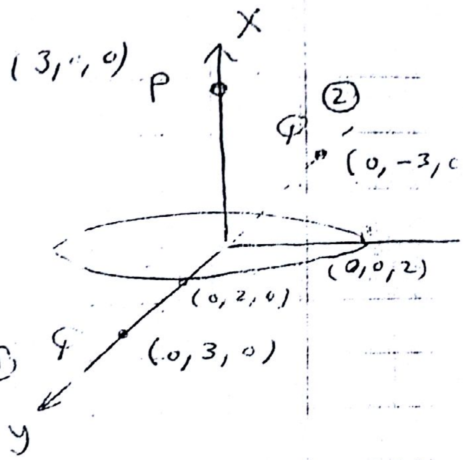
نصف القطر

$$\therefore \quad dE = \frac{\rho_L}{4\pi\epsilon_0} \frac{a\,d\phi}{r^3} \, x \, \bar{a}_x$$

$$\therefore \quad E_{Ring} = \frac{\rho_L \, a \, x}{4\pi\epsilon_0 \, (x^2+a^2)^{3/2}} \int_0^{2\pi} d\phi$$

$$= \frac{\rho_L}{2\epsilon_0} \frac{a\,x}{(x^2+a^2)^{3/2}} \bar{a}_x$$

$$= \frac{5\times10^{-6}}{\cancel{2}\,\epsilon_0} \frac{(\cancel{2})(3)}{(13)^{3/2}} \bar{a}_x = 3.6144 \ \text{V/m}$$



(b) :

$\therefore E_1 = \dfrac{1}{4\pi\epsilon_0} \dfrac{Q_1}{r_1^3} \bar{r}_1$

$\bar{r}_1 = (3, -3, 0)$

$|\bar{r}_1| = 3\sqrt{2}$

$r_2 = (3, 3, 0)$, $|r_2| = 3\sqrt{2}$

$E_1 = \dfrac{Q}{4\pi\epsilon_0 (3\sqrt{2})^3} (3\bar{a}_x - 3\bar{a}_y)$

$E_2 = \dfrac{Q}{4\pi\epsilon_0 (3\sqrt{2})^3} (3\bar{a}_x + 3\bar{a}_y)$

$\therefore E_T = 0 = \bar{E}_{Ring} + \bar{E}_1 + \bar{E}_2$

$0 = \left[3.6144 \times 10^4 + \dfrac{6Q}{4\pi\epsilon_0 (3\sqrt{2})^3} \right] \bar{a}_x$

$\therefore Q = 5.1115 \times 10^{-5} = 51.115$

$$\overline{r}_{line} = -\overline{a}_x - 4\,\overline{a}_z \quad , \quad |\overline{r}| = \sqrt{17}$$

$$\rho_L = 2 \times 10^{-9} \quad c/m$$

(c)

$$\overline{E} = \overline{E}_{line} + \overline{E}_{sheet} = \frac{\rho_L}{2\pi\epsilon_0 r}\,\overline{a}_r + \frac{\ }{2}$$

– لأنه ينقطع على يسار المستوى (الاحداثي أصغر من احداثي المستوى)

$$\therefore \overline{E} = \frac{2 \times 10^{-9}}{2\pi\epsilon_0\,(17)}\left(-\overline{a}_x - 4\,\overline{a}_z\right) + \frac{2 \times 10^{-9}}{2\epsilon_0}$$

$$= -115.058\,\overline{a}_x - 8.4591\,\overline{a}_z$$

$$|E| = \sqrt{(115.058)^2 + (8.4591)^2}$$

$$= 115.3685 \quad N/C$$

(b) :

direction of E at (4, 5, 6)

يقصد بها متجه الوحده $\overline{a}_E$

$\therefore \overline{a}_N = +\overline{a}_x$ احداثي x أكبر من المستوى

$$\overline{r} = 3\,\overline{a}_x + 2\,\overline{a}_z \quad , \quad |r| = \sqrt{13}$$

$$\therefore \overline{E} = \frac{2 \times 10^{-9}}{2\pi\epsilon_0\,(13)}\left(3\,\overline{a}_x + 2\,\overline{a}_z\right) + \frac{2 \times 10^{-9}}{2\epsilon_0}\,\overline{a}_x$$

$$\bar{E} = \square\ \bar{a}_x + \triangle\ \bar{a}_z$$

$$\therefore \bar{a}_E = \frac{\bar{E}}{|E|} = \frac{\square\ \bar{a}_x + \triangle}{\sqrt{\square^2 + \triangle^2}}$$

(c)

القوة ستكون بسبب المستوى على السلك:

$$F_{line} = Q\ E_{sheet}$$

$$F_{line} = \rho_L\ L\ E_{sheet}$$

$$\therefore \frac{F}{L} = \rho_L\ E_{sheet} = \rho_L\ \frac{\rho_s}{2\epsilon_0}\ (-$$

$1 = x_{line}$ لأن ال $\bar{a}_N = -\bar{a}_x$

$3 = x_{sheet}$

$$\therefore \frac{F}{L} = \frac{(2\times10^{-9})(2\times10^{-9})}{2\epsilon_0}\ -\bar{a}_x$$

$$= -\square\ \bar{a}_x$$

(d)

Q9 :

$Q = 100 \times 10^{-12}$ C at (4,

$\rho_L = 2 \times 10^{-9}$ c/m on x-axis

($y = 0$, $z = 0$)

$x = $ تتغير

$\rho_s = 5 \times 10^{-9}$ c/m^2 $z = 3$

$$E_{\text{point charge}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r_p^3} \bar{r}_p$$

$\bar{r}_p = (-3, 0, 4)$, $|\bar{r}_p| = \sqrt{9+16} = 5$

$\therefore \bar{E}_p = -0.0216 \, \bar{a}_x + 0.0288 \, \bar{a}_z$

$$\bar{E}_{line} = \frac{\rho_L}{2\pi\epsilon_0 r_L} \bar{a}_{r_L} = \frac{\rho_L}{2\pi\epsilon_0 r_L^2} \bar{r}_L$$

$\bar{r}_L = (0, 1, 1)$, $|r_L| = \sqrt{2}$

$$\therefore E_{line} = \frac{2 \times 10^{-9}}{2\pi\epsilon_0 (2)} (\bar{a}_y + \bar{a}_z)$$

$$= 18 \, \bar{a}_y + 18 \, \bar{a}_z$$

$$E_{sheet} = \frac{\rho_s}{2\epsilon_0} \bar{a}_N = \frac{5 \times 10^{-9}}{2 \epsilon_0} (-\bar{a}_z) = -28$$

$$\therefore \bar{E}_T = \bar{E}_p + \bar{E}_L + \bar{E}_s = \boxed{}$$

(4

Tanta University

Electrical Power and Machines Depart.

Electromagnetic Fields

Sheet (2)

Coulombs Law and Electric Field Intensity

Faculty of Engineering

1. A 20 nC point charge is located at P(2,4,-3) in free space. **Find**
   (a) E (r).
   (b) E at A (-3,2,0).
   (c) The locus of all points at which $E_x = 1\ V/m$.

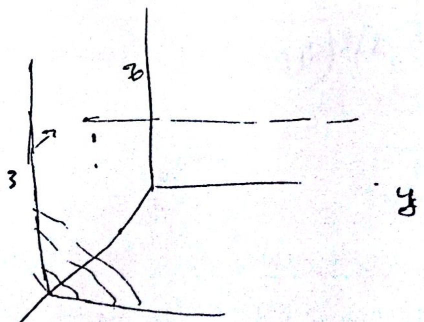
2. Two point charges of Q_1coulomb each are located at (0, 0, 1) and (0, 0,-1).**Determine** the locus of all possible positions of a third charge Q_2, where Q_2 may have any desired positive or negative value, such that the electric field intensity E = 0 at (0, 1, 0). **What** is the locus if the original charges are Q_1 and $-Q_1$?

3. A sheet of charges $\rho_s = 2\ nC/m^2$, is present at the plane $x = 3$ in free space, and a line charge $\rho_l = 2\ nC/m$ l is located at $x = 1, z = 4$.**Find**
   (a) The magnitude of electric field intensity at the origin.
   (b) The direction of E at p (4, 5, 6).
   (c) What is the force per meter length on the line charge?

4. A volume charge density $\rho_v = k/r$ ($r \neq 0$, and $k = a\ constant$) exists within a sphere of radius a. This charge distribution produces a certain electric field at $r > a$. **Determine**
   (a) The charge inside the sphere.
   (b) The value of a point charge placed at the origin which will produce the same field at $r > a$.

5. A straight wire L meter long is charged uniformly over one half of its length with a charge Q and over the other half of its length with a charge $-Q$. **Find** the electrostatic field at points along
   (a) The axis of wire. (b) The perpendicular bisector of wire.

6. A circular disk with radius a having a uniformsurface charge density $\rho_s\ C/m^2$**Determine** the electric field intensity at point P (0, 0, h) located on the axis of the disk.

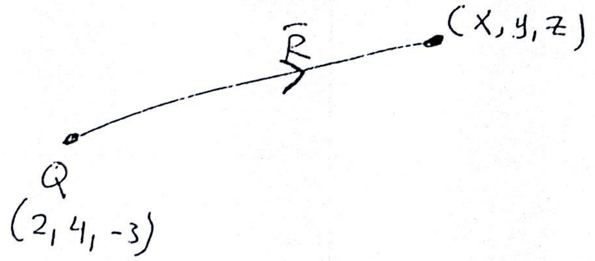


Sheet 2 Fields Coulomb's law

[1] $Q = 20\,nC \rightarrow P(2,4,-3)$

a) E_r

$$\bar{R} = (x-2)\,\bar{a}_x + (y-4)\,\bar{a}_y + (z+3)\,\bar{a}_z$$



$$|R| = \sqrt{\underbrace{(x-2)^2+(y-4)^2+(z+3)^2}} = \sqrt{\boxed{\ }}$$

$$E = \frac{Q}{4\pi\varepsilon_0 R^2}\,\bar{a}_R = \frac{Q}{4\pi\varepsilon_0 R^2}\,\frac{\bar{R}}{R} = \frac{Q}{4\pi\varepsilon_0 R^3}\,\bar{R}$$

$$\bar{E} = \frac{20\times10^{-9}}{4\pi\varepsilon_0\left(\boxed{\ }\right)^{3/2}}\left((x-2)\,\bar{a}_x + (y-4)\,\bar{a}_y + (z+3)\,\bar{a}_z\right)\ V/m$$

#

b) E at $(-3, 2, 0)$

بالتعويض في المعادلة السابقة

$x = -3$

$y = 2$

$z = 0$

$$\bar{E} = \frac{20\times10^{-9}}{4\pi\varepsilon_0\left((-5)^2+(-2)^2+(3)^2\right)^{3/2}}\left(-5\,\bar{a}_x - 2\,\bar{a}_y + 3\,\bar{a}_z\right)\ V/m$$

c) Locus of all points $E_x = 1\ V/m$

$$E_x = \frac{20 \times 10^{-9}}{4\pi\varepsilon_0 \left(\cancel{\ \ } \right)^{3/2}} (x-2) = 1 \quad \#$$

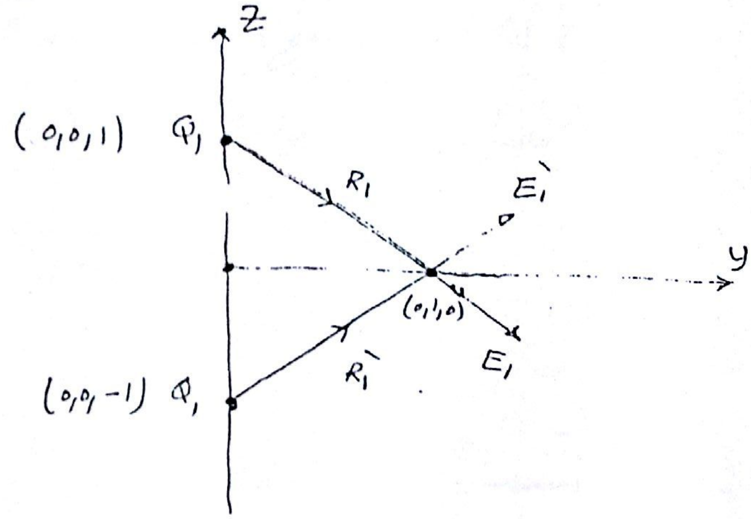
$\boxed{2}$

$$\bar{R}_1 = \bar{a}_y - \bar{a}_z$$

$$|R_1| = \sqrt{2}$$

$$\bar{R}_1' = \bar{a}_y + \bar{a}_z$$

$$|R_1'| = \sqrt{2}$$



$$E_1 + E_1' = \frac{Q_1}{4\pi\varepsilon_0 R_1^2} \bar{a}_{R_1} + \frac{Q_1}{4\pi\varepsilon_0 R_1'^2} \bar{a}_{R_2}$$

$$|R_1| = |R_1'|$$

$$E_1 + E_1' = \frac{Q_1}{4\pi\varepsilon_0 R_1^3} (\bar{R}_1 + \bar{R}_1')$$

$$E_1 + E_1' = \frac{Q_1}{4\pi\varepsilon_0 R_1^3} (2\bar{a}_y) = \frac{Q_1}{2\pi\varepsilon_0 R_1^3} \bar{a}_y$$

$$\boxed{E_1 + E_1' = \frac{Q_1}{4\pi\varepsilon_0 \sqrt{2}} \bar{a}_y}$$

3

الشحنة Q_2 يجب أن توضع بحيث تكون المجال الناتج عنها

يجعل E'_{tot} عند $(0,1,0)$ مساوي صفر

$$\therefore \quad |E_2| = |E_1 + E_1'|$$ المجالان متساويان

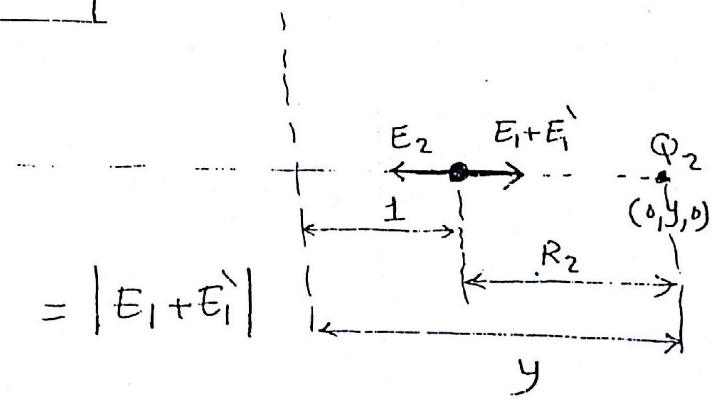
$$\bar{E}_2 = -(\bar{E}_1 + E_1')$$ عكس بعضهم

في الاتجاه وبعضها يجب أن تكون الشحنة Q_2 بحيث

تعطي مجال في اتجاه $(-a_y)$

Case 1 $\boxed{Q_2 \text{ is } +ve}$

Q_2 at $(0,y,0)$.



$$|R_2| = y-1$$

$$|E_2| = \frac{Q_2}{4\pi\varepsilon_0 R_2^2} = |E_1 + E_1'|$$

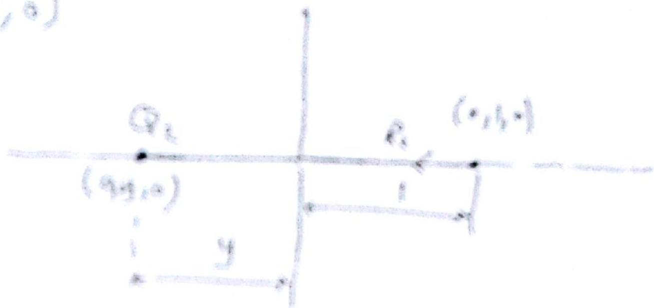
$$\frac{Q_2}{\cancel{4\pi\varepsilon_0}(y-1)^2} = \frac{Q_1}{\cancel{4\pi\varepsilon_0}\sqrt{2}}$$

$$(y-1)^2 = \frac{\sqrt{2}\,Q_2}{Q_1}$$

$$\therefore \quad \boxed{y = \sqrt{\frac{\sqrt{2}\,Q_2}{Q_1}} + 1} \quad \#$$

Case ② $\boxed{Q_2 \text{ is } -ve}$

Q_2 at $(0,-y,0)$



$|R_2| = y+1$

$$|E_2| = \frac{Q_2}{4\pi\varepsilon_o (y+1)^2}$$

$$\frac{Q_2}{4\pi\cancel{\varepsilon_o}(y+1)^2} = \frac{Q_1}{4\pi\varepsilon_o \sqrt{2}}$$

$$\boxed{y = \sqrt{\sqrt{2}\,\frac{Q_2}{Q_1}} - 1}$$

#

** الجزء الثاني في مسألة ال محلولة ع ←

(5)

$\boxed{3}$ $\rho_S = 2nC/m^2 \longrightarrow x=3$

$\rho_L = 2nC/m \longrightarrow x=1,\ z=4$

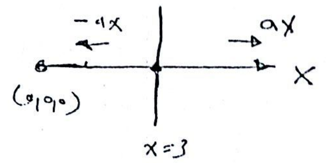
a) $|E|$ at $(0,0,0)$

$$\bar{E} = \bar{E}_s + \bar{E}_L$$

$$= \frac{\rho_s}{2\varepsilon_0} a_N + \frac{\rho_L}{2\pi\varepsilon_0 r} \bar{a}_r$$

$$a_N = -a_x$$

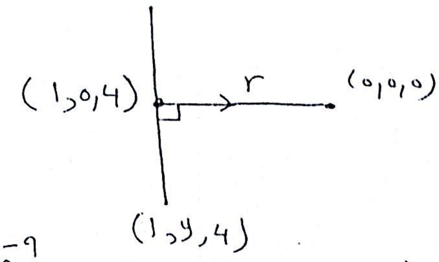
لأن احداثي النقطة (X) أصغر من احداثي السطح



$$\bar{r} = -\bar{a}_x - 4\bar{a}_z$$

$$|r| = \sqrt{17}$$

$$\bar{a}_r = \frac{-1}{\sqrt{17}}\bar{a}_x - \frac{4}{\sqrt{17}} a_z$$



$$\therefore \bar{E} = \frac{2*10^{-9}}{2\varepsilon_0}(-\bar{a}_x) + \frac{2*10^{-9}}{2\pi\varepsilon_0\sqrt{17}}\left(\frac{-1}{\sqrt{17}}\bar{a}_x - \frac{4}{\sqrt{17}}\bar{a}_z\right)$$

$$\bar{E} = \square\, \bar{a}_x + \triangle\, \bar{a}_z \quad \longleftarrow \text{مهم.}$$

$$|E| = \sqrt{\square^2 + \triangle^2} \quad \#$$

6

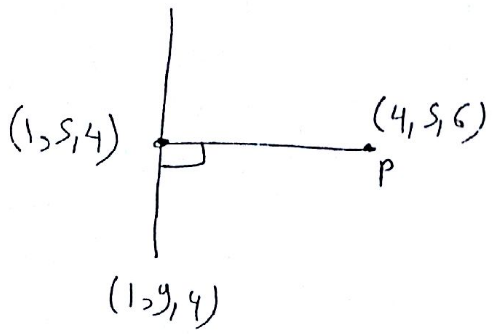
b) direction of $\bar{E}$ at $(4,5,6)$ $[\bar{a}_E]$

$$X_{point} = 4 > X_{surface} = 3 \longrightarrow a_N = +a_X$$

$$\bar{r} = 3\bar{a}_x + 2\bar{a}_z$$

$$|r| = \sqrt{13}$$

$$\bar{a}_r = \frac{3}{\sqrt{13}}\bar{a}_x + \frac{2}{\sqrt{13}}\bar{a}_z$$



$$\bar{E} = \frac{2*10^{-9}}{2\varepsilon_0}\bar{a}_x + \frac{2*10^{-9}}{2\pi\varepsilon_0\sqrt{13}}\left(\frac{3}{\sqrt{13}}\bar{a}_x + \frac{2}{\sqrt{13}}\bar{a}_z\right)$$

$$\bar{E} = \boxed{\equiv}\,\bar{a}_x + \triangle\,\bar{a}_z$$

$$\bar{a}_E = \frac{\boxed{\equiv}\,\bar{a}_x + \triangle\,\bar{a}_z}{\sqrt{\boxed{\equiv}^2 + \triangle^2}}$$ #

c) Force per unit length on the line charge.

$$F = Q\,E_s = (\rho_L\,L)\,E_s$$

Force per unit length $= \frac{F}{L} = \rho_L E_s = \rho_L \frac{\rho_s}{2\varepsilon_0} a_N$

$$X_{line} = 1 < X_{surface} = 3 \rightarrow a_N = -a_X$$

$$\therefore \frac{\bar{F}}{L} = \frac{\rho_L\rho_s}{2\varepsilon_0}(-\bar{a}_x)\ N/m$$ #

4. $\rho_v = \frac{k}{r}$ $\quad K = a$

$\therefore \rho_v = \frac{a}{r}$

a) Q within the sphere

نظراً لأن ρ_v ليست قيمة ثابتة
يتم ايجاد Q باستخدام التكامل

$$Q = \int \rho_v \, dv = \int_{vol.} \frac{a}{\cancel{r}} r^{\cancel{2}} \sin\theta \, dr \, d\theta \, d\phi$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{a} a * r \sin\theta \, dr \, d\theta \, d\phi$$

$$= a * \frac{r^2}{2}\Big|_0^a * -\cos\theta\Big|_0^{\pi} * \phi\Big|_0^{2\pi}$$

$$= \frac{a^3}{2} * 2 * 2\pi = 2\pi a^3 \text{ Colo.} \#$$

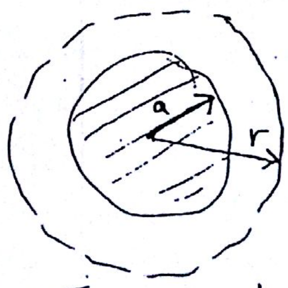
b) $Q_1 = ?$ $\longrightarrow$ placed at $(0,0,0)$ $\quad r > a$

باستخدام قانون جاوس من الفصل الثالث

$$\oint D \cdot ds = Q_{enc}$$

$$D(4\pi r^2) = 2\pi a^3$$

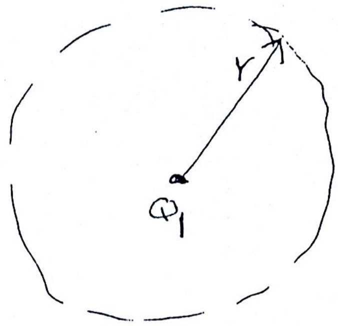
$$D = \frac{2\pi a^3}{4\pi r^2} \quad (1)$$



سطح جاوس هو كرة نصف قطرها r

the pointcharge Q_1

سطح جاوس كرة نصف قطرها (r)



$$\oint D \cdot ds = Q_{enc}$$

$$D\,(4\pi r^2) = Q_1$$

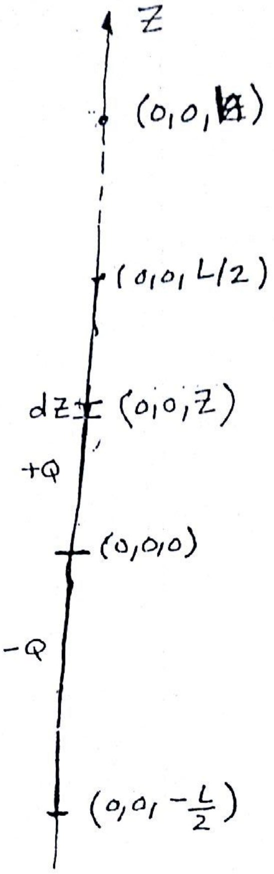
$$D = \frac{Q_1}{4\pi r^2} \quad (2)$$

لأن نعطي نفس المجال في الحالتين

$$(1) = (2)$$

$$\frac{Q_1}{4\pi r^2} = \frac{2\pi a^3}{4\pi r^2} \rightarrow \therefore \boxed{Q_1 = 2\pi a^3\, C}$$

#



Sheet 2 (P5)

$+Q \leftarrow$ نصف الخط

$-Q \leftarrow$ نصف الخط

a) at $(0,0,h)$

عاملهم على أنهم خطين.

$(0 \rightarrow \frac{L}{2})$

$$\bar{R} = (h-Z)\,\bar{a}_Z$$

$$|R| = h-Z$$

$$\bar{a}_R = \bar{a}_Z$$

$$dQ = \rho_L \, dL = \rho_L \, dZ$$

$$\rho_L = \frac{Q}{\frac{L}{2}} = \frac{2Q}{L} \qquad \therefore\ dQ = \frac{2Q}{L}\, dZ$$

$$d\bar{E}_1 = \frac{dQ}{4\pi\varepsilon_0 R^2}\,\bar{a}_R = \frac{2Q/L \; dZ}{4\pi\varepsilon_0 (h-Z)^2}\,\bar{a}_Z$$

$$E_1 = \int \frac{\cancel{2}Q/L}{2\cancel{4}\pi\varepsilon_0 (h-Z)^2}\, dZ \quad \bar{a}_Z$$

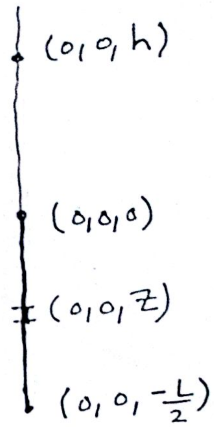
$$= \frac{Q}{2\pi\varepsilon_0 L}\int_0^{L/2} (h-Z)^{-2}\, dZ \quad \bar{a}_Z$$

$$\bar{E}_1 = \frac{Q}{2\pi\varepsilon_0 L}\;\frac{1}{(h-Z)}\Bigg|_0^{L/2} = \frac{Q}{2\pi\varepsilon_0 L}\left\{\frac{1}{h-\frac{L}{2}} - \frac{1}{h}\right\}$$

$(-\frac{L}{2} \rightarrow 0)$

تم فرض النقطة $(0,0,Z)$

و حدود التكامل ستكون سالب



$$\rho_L = \frac{-Q}{L/2} = \frac{-2Q}{L}$$

$$\bar{R} = (h-z)\,\bar{a}_z \qquad |R| = h-z$$

$$\bar{a}_R = \bar{a}_z$$

$$dQ = \rho_L\, dz = \frac{-2Q}{L}\, dz$$

$$d\bar{E}_2 = \frac{-2Q/L \quad dz}{4\pi\varepsilon_0 (h-z)^2}\,\bar{a}_z = \frac{-Q}{2\pi\varepsilon_0 L}\,\frac{dz}{(h-z)^2}\,\bar{a}_z$$

$$\bar{E}_2 = \int d\bar{E}_2 = \frac{-Q}{2\pi\varepsilon_0 L}\int_{-\frac{L}{2}}^{0}\frac{dz}{(h-z)^2}\,\bar{a}_z$$

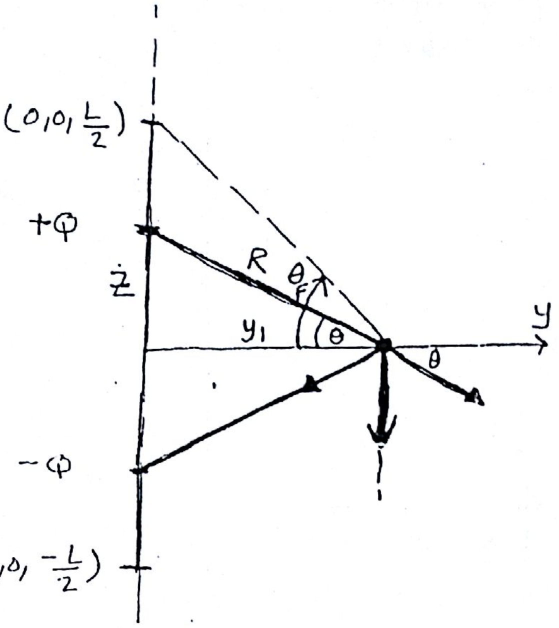
$$= \frac{-Q}{2\pi\varepsilon_0 L}\,\frac{1}{h-z}\Big|_{-L/2}^{0} = \frac{-Q}{2\pi\varepsilon_0 L}\left\{\frac{1}{h} - \frac{1}{h+\frac{L}{2}}\right\}\bar{a}_z$$

$$\bar{E}_{(0,0,h)} = \bar{E}_1 + \bar{E}_2 = \frac{Q}{2\pi\varepsilon_0 L}\left\{\frac{1}{h-\frac{L}{2}} - \frac{1}{h} - \frac{1}{h} + \frac{1}{h+\frac{L}{2}}\right\}\bar{a}_z$$

$$\bar{E} = \frac{Q}{2\pi\varepsilon_0 L}\left\{\frac{1}{h-\frac{L}{2}} + \frac{1}{h+\frac{L}{2}} - \frac{2}{h}\right\}\bar{a}_z \quad \#$$

b) at $(0, y_1, 0)$

من التماثل سيكون المجال
في اتجاه $(-\bar{a}_z)$ فقط



$$dE = \frac{dQ}{4\pi\varepsilon_0 R^2}\,\bar{a}_R$$

$$dE = \frac{2\,dQ}{4\pi\varepsilon_0 R^2}\sin\theta\,(-\bar{a}_z)$$

$$dQ = \rho_L\,dz = \frac{2Q}{L}\,dz$$

$$\cos\theta = \frac{y_1}{R} \rightarrow R = y_1\sec\theta$$

$$\tan\theta = \frac{z}{y_1} \rightarrow z = y_1\tan\theta \rightarrow dz = y_1\sec^2\theta\,d\theta$$

$$\therefore\; dE = \frac{\cancel{2}\,\frac{\cancel{2}Q}{L}\,\cancel{y_1}\,\cancel{\sec^2\theta}\,d\theta}{\cancel{4}\pi\varepsilon_0\,y_1^{\cancel{2}}\,\cancel{\sec^2\theta}}\sin\theta\,(-\bar{a}_z)$$

$$dE = \frac{Q}{\pi\varepsilon_0 L y_1}\sin\theta\,d\theta\,(-\bar{a}_z)$$

$$\bar{E} = \int dE = \frac{Q}{\pi\varepsilon_0 L y_1}(-\cos\theta)\Big|_0^{\theta_F} = \frac{Q}{\pi\varepsilon_0 L y_1}\left\{1-\cos\theta_F\right\}$$

$$\bar{E} = \frac{Q}{\pi\varepsilon_0 L y_1}\left\{1 - \frac{y_1}{\sqrt{y_1 + (\frac{L}{2})^2}}\right\}(-\bar{a}_z) \quad \#$$

Scanned by CamScanner

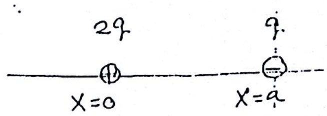
2- A charge q_1=2q is at the origin and a charge q_2=-q is on the x-axis at x=a. Find expressions for the total electric field E on the x-axis in each of the regions a) x <0; b) 0< x <a, c) x>a. d) Determine all points on the x axis where the electric field is zero e) Make a plot of E_X versus x on the x axis, -∞ < x <∞.

Solution

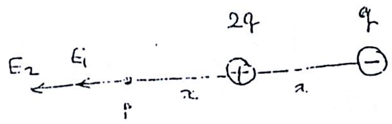
$q_1 = 2q \longrightarrow (0,0,0)$

$q_2 = -q \longrightarrow (a,0,0)$

$E = ? \longrightarrow (x,0,0)$



a) $x < 0$

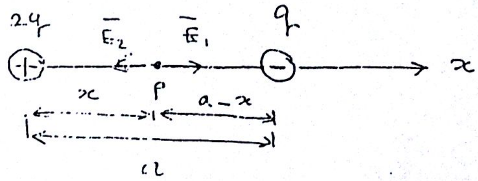


$$\bar{E} = \bar{E}_1 + \bar{E}_2$$

$$= \frac{2q}{4\pi\varepsilon_0 x^2}(-\bar{a}_x) + \frac{-q}{4\pi\varepsilon_0(x+a)^2}(-\bar{a}_x)$$

$$\bar{E} = \frac{q}{4\pi\varepsilon_0}\left\{\frac{2}{x^2} - \frac{1}{(a+x)^2}\right\}(-\bar{a}_x)$$

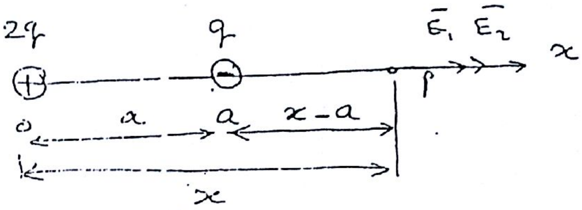
b) $0 < x < a$



$$\bar{E} = \frac{2q}{4\pi\varepsilon_0 x^2}\bar{a}_x + \frac{-q}{4\pi\varepsilon_0(a-x)^2}(-\bar{a}_x)$$

$$\bar{E} = \frac{q}{4\pi\varepsilon_0}\left\{\frac{2}{x^2} + \frac{1}{(a-x)^2}\right\}\bar{a}_x$$

(c) $\underline{x > a}$



$$\bar{E} = \frac{2q}{4\pi\varepsilon_0 x^2}\bar{a}_x + \frac{-q}{4\pi\varepsilon_0 (x-a)^2}\bar{a}_x$$

$$= \frac{q}{4\pi\varepsilon_0}\left\{\frac{2}{x^2} - \frac{1}{(x-a)^2}\right\}\bar{a}_x$$

$$\{\bar{E}\} = \begin{cases} \frac{q}{4\pi\varepsilon_0}\left(-\frac{2}{x^2} + \frac{1}{(a+x)^2}\right)\bar{a}_x & x < 0 \\ \frac{q}{4\pi\varepsilon_0}\left(\frac{2}{x^2} + \frac{1}{(a-x)^2}\right)\bar{a}_x & 0 < x < a \\ \frac{q}{4\pi\varepsilon_0}\left(\frac{2}{x^2} - \frac{1}{(x-a)^2}\right)\bar{a}_x & x > a \end{cases}$$

(d)

$E = 0$ at $x = ??$

for $\underline{x < 0}$ $\quad \frac{2}{x^2} = \frac{1}{(a+x)^2} \Longrightarrow$ $x = -0.583\,a$, $x = -3.414\,a$

for $0 < x < a$

$$\frac{2}{x^2} = \frac{-1}{(a-x)^2} \quad - x^2 = 2\ldots$$

$$2(a-x)^2 = -x^2$$

$$2a^2 - 4aX + 2X^2 = -X^2$$

$$3x^2 - 4aX + 2a^2 = 0$$

المميز $= \sqrt{B^2 - 4AC} = \sqrt{16a^2 - 4 \times 3 \times 2a^2} = \sqrt{-8a^2} \rightarrow$ imajnary لا يوجد حل

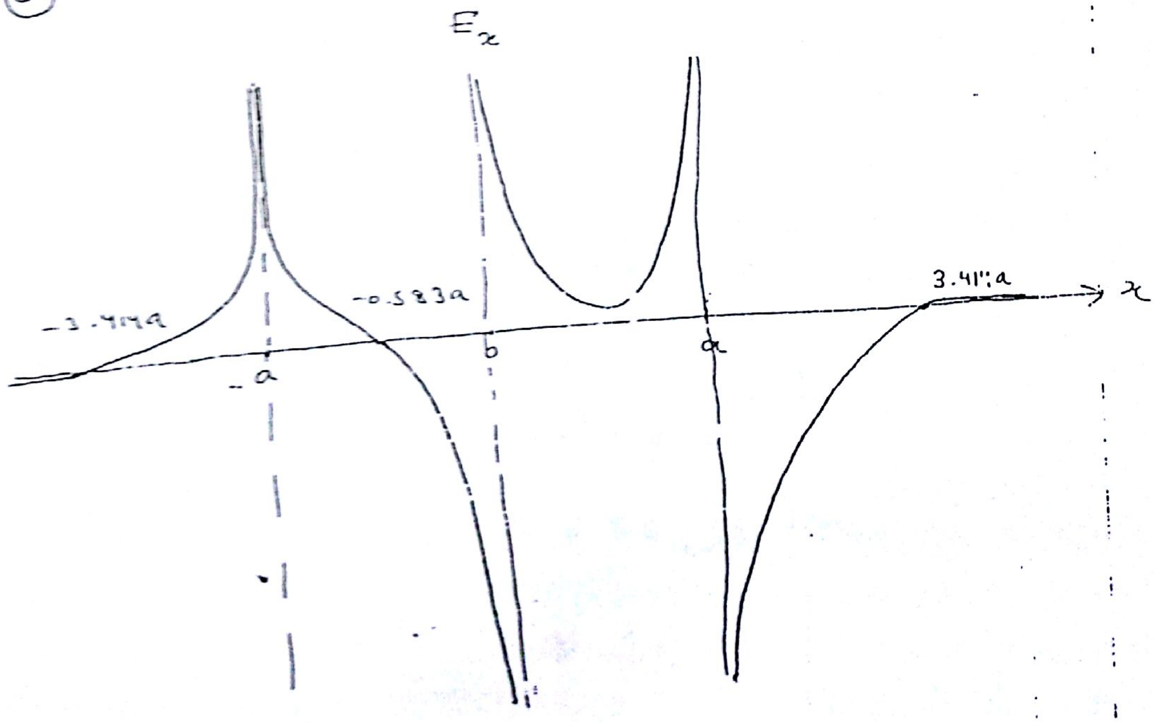
for $x > a$

$$\frac{2}{x^2} = \frac{1}{(x-a)^2} \rightarrow X^2 - 4aX + 2a^2 = 0$$

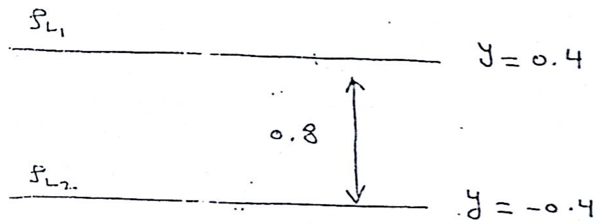
$$x = 3.414a = (2+\sqrt{2})a$$

$X = 0.59a \longrightarrow$ refused $\boxed{\text{لأنه أقل من (a)}}$

(e)



3- Two identical uniform line charges of 75 nC/m are located in free space at x=0, y= ± 0.4 m. What force per unit length does each line charge exert on the other?



$F/l = ?$

$F = QE$

$F = \rho_L l E \longrightarrow \frac{\rho_L}{2\pi\varepsilon_0 r}$

$$F/l = \frac{\rho_L^2}{2\pi\varepsilon_0 r}$$

$$= \frac{(75 \times 10^{-9})^2}{2\pi \frac{10^{-9}}{36\pi}(0.8)}$$

$= \cancel{75^2 \times} \; .8$

$= 126.56 \; \mu \; N/m$

\#

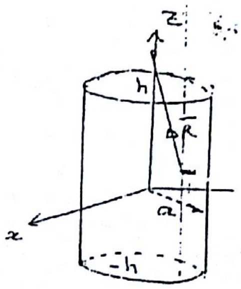
8- A uniform surface charge density ρ_s is distributed over a cylindrical surface of r=a extending from z=-h to z=h. Find the electrical field intensity in free space at (0,0,k).

$$\bar{E} = \int_s \frac{\rho_s \, ds}{4\pi \varepsilon_0 R^2} \bar{a}_R$$

$$ds = r \, d\phi \, dz = a \, d\phi \, dz$$

$$0 \leq \phi \leq 2\pi$$

$$-h \leq z \leq h$$



$$\bar{R} = (0,0,K) - (x,y,z)$$

$$= -x\bar{a}_x - y\bar{a}_y + (K-z)\bar{a}_z$$

$$= -a\,\bar{a}_r + (K-z)\bar{a}_z$$

$$R = \sqrt{a^2 + (K-z)^2}$$

$$\bar{E} = \int_{-h}^{h}\int_{0}^{2\pi} \frac{\rho_s \, a \, d\phi \, dz}{4\pi \varepsilon_0 \left(a^2 + (K-z)^2\right)^{3/2}} \left(\cancel{-a\bar{a}_r} + (K-z)\bar{a}_z\right)$$

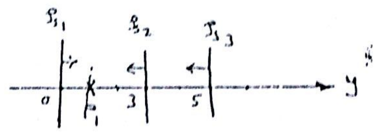
cancelled due to symmetry

$$= \frac{\rho_s a}{4\pi \varepsilon_0} 2\pi \int_{-h}^{h} \frac{(K-z)\,dz}{\left[a^2 + (K-z)^2\right]^{3/2}} \bar{a}_z$$

$$= \frac{\rho_s a}{2 \varepsilon_0} \left[\frac{1}{\sqrt{a^2 + (K-h)^2}} - \frac{1}{\sqrt{a^2 + (K+h)^2}} \right] \bar{a}_z$$

#

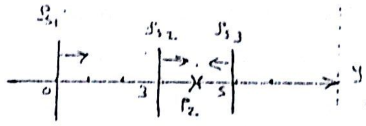
9- Infinite charged plane sheets of charge lie in the y=0, y=3, and y=5 planes respectively. Given that the net electric field intensities at (1,1,1), (2,4,2) and (4,6,8) are 0.0, 0.8 and 6.0 V/m all in y direction. Find the surface charge densities and the electric field intensity at P(3,-2,5)



<u>at P_1</u>

$$\bar{E}_{P_1} = \frac{\rho_{s1}}{2\varepsilon_0}\bar{a}_y - \frac{\rho_{s2}}{2\varepsilon_0}\bar{a}_y - \frac{\rho_{s3}}{2\varepsilon_0}\bar{a}_y = 0$$

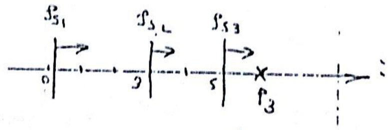
$$\rho_{s1} - \rho_{s2} - \rho_{s3} = 0 \longrightarrow (1)$$



<u>at P_2</u>

$$\bar{E}_{P_2} = \frac{\rho_{s1}}{2\varepsilon_0}\bar{a}_y + \frac{\rho_{s2}}{2\varepsilon_0}\bar{a}_y - \frac{\rho_{s3}}{2\varepsilon_0}\bar{a}_y = 0.8\,\bar{a}_y$$

$$\rho_{s1} + \rho_{s2} - \rho_{s3} = 1.6\,\varepsilon_0 \longrightarrow (2)$$



<u>at P_3</u>

$$\bar{E}_{P_3} = \frac{\rho_{s1}}{2\varepsilon_0}\bar{a}_y + \frac{\rho_{s2}}{2\varepsilon_0}\bar{a}_y + \frac{\rho_{s3}}{2\varepsilon_0}\bar{a}_y = 6\,\bar{a}_y$$

$$\rho_{s1} + \rho_{s2} + \rho_{s3} = 12\,\varepsilon_0 \longrightarrow (3)$$

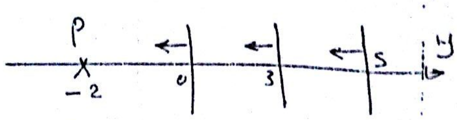
$$\rho_{s1} = 6\,\varepsilon_0 \quad C/m^2$$

$$\rho_{s2} = 0.8\,\varepsilon_0 \quad C/m^2$$

$$\rho_{s3} = 5.2\,\varepsilon_0 \quad C/m^2$$

<u>at P(3,-2,5)</u>

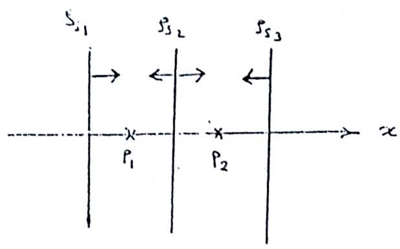
$y = -2$



$$\bar{E} = \frac{-\rho_{s1}}{2\varepsilon_0}a_y - \frac{\rho_{s2}}{2\varepsilon_0}a_y - \frac{\rho_{s3}}{2\varepsilon_0}a_y$$

$$\bar{E} = -6\,\bar{a}_y \ \#$$

10- Three infinite uniformly charged sheets are arranged in parallel. Is it possible to find the electric field to be zero in both regions between sheets or not?



at P_1

$$|\vec{E}| = \frac{\rho_{s1}}{2\varepsilon_0} - \frac{\rho_{s2}}{2\varepsilon_0} - \frac{\rho_{s3}}{2\varepsilon_0} = 0$$

$$\boxed{\rho_{s1} - \rho_{s2} - \rho_{s3} = 0} \rightarrow (1)$$ Condition for region I

at P_2

$$|\vec{E}| = \frac{\rho_{s1}}{2\varepsilon_0} + \frac{\rho_{s2}}{2\varepsilon_0} - \frac{\rho_{s3}}{2\varepsilon_0} = 0$$

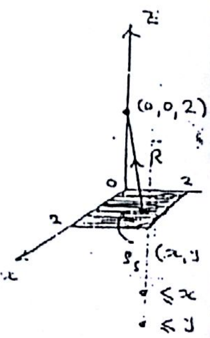
$$\boxed{\rho_{s1} + \rho_{s2} - \rho_{s3} = 0} \rightarrow (2)$$ Condition for region II

It's impossible to find E to be zero in both regions I and II.

#

18- A finite sheet of charge, of density $\rho_s = 2x(x^2+y^2+4)^{3/2}$ nC/m2, lies in the z = 0 plane for $0 \le x \le 2$ m and $0 \le y \le 2$ m. Determine E at (0, 0, 2) m.

$$\bar{E} = \int_s \frac{\rho_s\, ds}{4\pi\varepsilon_0 R^2}\,\bar{a}_R$$



$ds = dx\,dy$ $\quad 0 \le x \le 2$, $\quad 0 \le y \le 2$

$\bar{R} = (0,0,2) - (x,y,0) = (-x,-y,2)$

$R = \sqrt{x^2+y^2+4}$

$$\therefore\ \bar{E} = \int_0^2\int_0^2 \frac{2x(\cancel{x^2+y^2+4})^{\cancel{3/2}}\,dx\,dy}{4\pi\varepsilon_0(\cancel{x^2+y^2+4})^{\cancel{3/2}}}(-x,-y,2)$$

$$= \frac{10^{-9}}{2\pi\varepsilon_0}\left\{\int_0^2\int_0^2 -x^2\,dx\,dy\,(\bar{a}_x) + \int_0^2\int_0^2 -xy\,dx\,dy\,(\bar{a}_y) + \int_0^2\int_0^2 2x\right.$$

$$= 18\times\cancel{10^{+9}}\times\cancel{10^{-9}}\left\{-2\,\frac{2^3}{3}\,\hat{a}_x - \frac{\cancel{2}^2}{\cancel{2}}\,\frac{2^2}{\cancel{2}}\,\bar{a}_y + 2(\cancel{2})\,\frac{2^2}{\cancel{2}}\,\bar{a}_z\right\}$$

$$= -96\,\bar{a}_x - 72\,\bar{a}_y + 144\,\bar{a}_z \quad \text{V/m}$$

#